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Einstein's Equations near Spatial Infinity*

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Abstract. A new class of space-times is introduced which, in a neighbourhood of spatial infinity, allows an expansion in negative powers of a radial coordinate. Einstein's vacuum equations give rise to a hierarchy of linear equations for the coefficients in this expansion. It is demonstrated that this hierarchy can be completely solved provided the initial data satisfy certain constraints.

1. Introduction and Motivation

Minkowski space, the arena of special relativity, has a much richer structure "at infinity" than flat Riemannian spaces. One can move to infinity along timelike, spacelike and null lines, which cannot be mapped into each other by Poincaré transformations. Correspondingly one can investigate the behaviour of fields on Minkowski space in different asymptotic regions.

Light cones, or more generally null hypersurfaces, are characteristic hypersurfaces for hyperbolic equations, constructed geometrically from the Minkowski metric. (The scalar-wave equation, Maxwell's equations and the Yang–Mills equations are important examples.) The radiation contained in such fields propagates to infinity along the bicharacteristics of such hypersurfaces, i.e. null lines. For this reason, Bondi et al. [1] in their studies of radiation considered expansions of the type

$$\Phi = \frac{{}^0\psi(u,\theta,\phi)}{r} + \frac{{}^1\psi(u,\theta,\phi)}{r^2} + \cdots, \tag{1.1}$$

where $u = t - r$ and (r, θ, ϕ) are standard polar coordinates. If Φ satisfies the scalar-wave equation $\square\Phi = \eta^{\mu\nu}\Phi_{,\mu\nu} = 0 (\eta^{\mu\nu} = \mathrm{diag}(- + + +)$, $\mu,\nu = 0,1,2,3)$, one gets a recursion relation for the coefficients of the expansion (1.1)

$$-2n\frac{\partial^n\psi}{\partial u} = \{L^2 + n(n-1)\}^{n-1}\psi, \quad n \geqq 1, \tag{1.2}$$

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