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Statistical Mechanics of Vortices in an Inviscid Two-Dimensional Fluid

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Abstract. In this paper we study rigorously the statistical mechanics of a gas of vortices in the thermodynamic limit. In this limit, no negative temperature states are found to exist.

0. Introduction

When the motion of a fluid may be considered as approximately two-dimensional, the fluid often exhibits well-defined vortices. The existence of such vortices is an experimental fact, which Poincaré already tried to explain [1]. If dissipation may be neglected, the motion of the vortices is Hamiltonian, and it is natural to study the "gas of vortices" by the methods of statistical mechanics. Onsager [20] has argued that when the total energy of the system is sufficiently large, the "gas of vortices" is in a "negative temperature state." He further argued that, in such a state, vortices of the same sign attract each other. In fact the coalescence of vortices of the same sign has later been observed in computer experiments (see Montgomery and Joyce [17]), and is claimed to explain in part the existence of large well-defined vortices [2].

In this paper we study rigorously the statistical mechanics of a gas of vortices in the thermodynamic limit. Thus we let the volume of fluid go to infinity, while the density and mean energy of vortices tend to constants. In this limit, no negative temperature states are found to exist [3], contrary to Onsager's proposal.

Our main results are presented in Sect. 3 (Theorems 3.1 and 3.2).

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1 The argument of Poincaré [22, Chap. VIII] is based on a discussion of the stability of motion

2 See also Kraichnan and Montgomery [15] for a discussion of this theory. Note that vorticity is conserved in an inviscid fluid (theorem of Helmholtz); therefore Onsager's mechanism cannot explain the appearance of well defined vortices in a fluid where the vorticity is smoothly spread out initially

3 Negative temperature states are known to exist for certain other systems without kinetic energy (spin systems). We claim that nothing of the sort is present here

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