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The Existence of Constant Mean Curvature Foliations of Gowdy 3-Torus Spacetimes*

James Isenberg[1] and Vincent Moncrief[2]

1 Department of Mathematics, University of California, Berkeley, CA 94720, USA

2 Department of Physics, Yale University, New Haven, CT 06511, USA

Abstract. We consider the class of smooth, maximally extended, globally hyperbolic, vacuum, Gowdy spacetimes on $T^3 \times R$ and prove that these spacetimes are globally foliated by space-like, constant mean curvature hypersurfaces. Our results can easily be extended to cover electrovac solutions of the same symmetry type and can probably be extended to cover other spacetime topologies as well.

I. Introduction

The importance of constant mean curvature (CMC) hypersurfaces as useful tools for studying the physics of general relativistic spacetimes is widely recognized. Nevertheless little is yet known about the class of spacetimes which admit them. It has been conjectured that every maximally extended, globally hyperbolic, spatially compact solution of Einstein's equations (in vacuum or with "reasonable" source field coupling) can be foliated by CMC hypersurfaces [1, 2]. However this conjecture is known to be true only for a handful of examples such as the spatially homogeneous cosmological models [3]. In this paper we shall extend the validity of the conjecture to a rather wide class of spacetimes—the Gowdy models [4] with $T^3 \times R$ topology. For simplicity we consider only vacuum spacetimes. However, one could easily extend our result to Einstein–Maxwell spacetimes of the same symmetry type.

This paper is not intended to provide a review of the known results concerning existence and uniqueness of CMC hypersurfaces; nor does it discuss the usefulness of such hypersurfaces once they have been obtained. (For the former we refer the reader to the article by Marsden and Tipler [5]; for the latter we suggest the 1979 Battelle conference proceedings edited by Smarr [6]). Rather, the paper is aimed directly at proving our result. We proceed by first defining the Gowdy metrics and then stating and proving our theorem. We conclude with a discussion of the possible

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