Commun. Math. Phys. 86, 391–418 (1982)

Communications in Mathematical Physics

Calculation of Norms of Bethe Wave Functions

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Abstract. A class of two dimensional completely integrable models of statistical mechanics and quantum field theory is considered. Eigenfunctions of the Hamiltonians are known for these models. Norms of these eigenfunctions in the finite box are calculated in the present paper. These models include in particular the quantum nonlinear Schrödinger equation and the Heisenberg XXZ model.

1. Introduction

A lot of two dimensional models have been solved by means of the Bethe Ansatz, see for example [1–5]. The recently formulated quantum inverse scattering method (QISM) [6, 7] disclosed the algebraic nature of these solutions. An interesting problem is the study of perturbations of these models. Such a study requires the knowledge of the norms of eigenfunctions in the finite box. Gaudin studied this problem for the quantum nonlinear Schrödinger equation [8] and made the remarkable hypothesis that the norm of the eigenfunction is equal to some Jacobian. A similar formula for the norm of the eigenfunction in the Heisenberg XXZ model was presented in [9] by Gaudin et al. Authors of [9] emphasized that the arguments given in their paper do not really constitute a proof. In the present paper these formulae are proved and a more general result is obtained. The norms are calculated for any exactly solvable models with the R matrix either of the XXX model or of the XXZ Heisenberg models.

The contents of the paper are as follows. In Sect. 2 the main formulas of QISM are presented. The proof of the norm formula is long, so in Sects. 3 and 4 the final formulas are given. In Sect. 5 the explicit expression for the anticipated answer is examined. We prove that this expression is characterized uniquely by some of its properties. In the rest of the paper these properties are proved for the norms of Bethe wave functions themselves. As the calculation of the norm itself is

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0010-3616/82/0086/0391/$05.60