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The Existence of a Non-Minimal Solution to the SU(2) Yang-Mills-Higgs Equations on $\mathbb{R}^3$: Part II

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Abstract. This paper proves that there exists a finite action solution to the SU(2) Yang-Mills-Higgs equations on $\mathbb{R}^3$ in the Bogomol'nyi-Prasad-Sommerfield limit which is not a solution to the first order Bogonol'nyi equations. The existence is established using Ljusternik-Šnirelman theory on non-contractible loops in the configuration space.

I. Introduction

In the first paper in this series ([1], to be referred to as Part I), the author stated the following theorem:

Theorem 1.1. *There exists a smooth, finite action solution to the* SU(2) *Yang-Mills-Higgs equations in the Bogomol'nyi-Prasad-Sommerfield limit which does not satisfy the first order Bogomol'nyi equations.*

This sequel to Part I contains the proof of Theorem 1.1. The reader is referred to Sects. I.2, 3 for an introduction to Yang-Mills-Higgs theory. These sections also define the author's terminology and notation.

The proof of Theorem 1.1 is an application of Ljusternik-Šnirelman theory on the space of finite action field configurations with monopole number zero (denoted $\mathscr{C}_0$). Part I established that a solution to the Yang-Mills-Higgs equations (I.2.2, 3) with non-zero action exists in $\mathscr{C}_0$ if there exists $k>0$, and a non-trivial generator $e\in \Pi_k(\mathrm{Maps}((S^2,n);(S^2,n),e_*)$ such that

$$\inf_{c(y)\in\Lambda(e)}\left\{\sup_{y\in S^k}\mathscr{a}(c(y))\right\}<8\pi. \tag{1.1}$$

Such a solution cannot satisfy the Bogomol'nyi equations (I.2.6). It is the purpose of this paper to establish that the above criteria is satisfied and Sects. 2–5 prove that (1.1) is satisfied for the generator of $\Pi_1(\mathrm{Maps}((S^2,n);(S^2,n)),e_*)$. It is also

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