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Dobrushin Uniqueness Techniques and the Decay of Correlations in Continuum Statistical Mechanics

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Abstract. Uniqueness of Gibbs states and decay properties of averaged, two point correlation functions are proved for many-body potentials in continuum statistical mechanical models via Dobrushin uniqueness techniques.

Introduction

Gross [6], using Dobrushin uniqueness techniques [1], has given decay rates for two point correlation functions in classical lattice models at high temperature or low activity. This paper extends those techniques to the continuum case and gives sufficient conditions on physically reasonable continuum potentials for the analogous results to hold along with uniqueness of Gibbs states. The models studied here are based on the same measurable space used by Preston [12] and Ruelle [14] in their studies of Gibbs states. Our results rest on the assumption that the set of Gibbs states for the models we consider is non-empty at high temperature. This has been shown to be true in the case of pair potentials by Ruelle [14].

Section 2 of the paper extends the results of Gross [6] to the continuum case with necessary added hypotheses. Section 3 gives conditions on potentials for these hypotheses to be satisfied.

Section 1. Notations and Definitions

Let Λ be a bounded Borel set in $\mathbb{R}^d$. We take $(X(\Lambda), B_\Lambda)$ to denote the measurable space of configurations of particles in Λ described in Preston [12], and $X_N(\Lambda)$ denotes the configurations of cardinality N in Λ. Let Ω be the set of locally finite subsets of $\mathbb{R}^d$, representing configurations of particles in $\mathbb{R}^d$. We will let $\Omega_F \subset \Omega$ denote the subsets of finite cardinality in Ω and $|s|$ denote the cardinality of $s \in \Omega_F$. S is the σ-algebra on Ω generated by sets of the form $\{s \in \Omega : |s \cap B| = m\}$, where B runs over bounded Borel sets of $\mathbb{R}^d$ and m runs over the set of non-negative integers.

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