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On the Symplectic Structure of General Relativity

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Abstract. The relation between the symplectic structures on the canonical and radiative phase spaces of general relativity is exhibited.

1. Introduction

There are available in the literature, two Hamiltonian descriptions of general relativity. The first and the more established one is based on spacelike hypersurfaces and uses the initial value formulation of general relativity and the Dirac theory of constrained systems [1, 2]. Over the years, this formulation has been systematically developed and refined by several authors and has shed considerable light on the structure of Einstein's theory. (See, e.g., [3].) In particular, these investigations have brought out the role of the Arnowitt–Deser–Misner [4] energy-momentum as the generator of space-time translations [5] and have paved the way for canonical quantization of gravity [3]. The second Hamiltonian description became available more recently [6]. It is based on null infinity [7] and uses techniques from the gravitational radiation theory in exact general relativity. (See especially, [8] and [9].) Here the focus is on the radiative aspects of the gravitational field; the phase space is the space of radiative modes. This description has also given one new insight. In particular, fluxes of energy-momentum and angular momentum carried away by gravitational waves have been shown to be the generators of the Bondi–Metzner–Sachs (BMS) group, the asymptotic symmetry group at null infinity [10]. More importantly, the formulation has enabled one to carry out the asymptotic quantization of the non-linear gravitational field [6, 11].

In view of this situation, it is natural to ask for the relation between the two descriptions. Apart from its intrinsic interest, such an analysis would clarify several issues which arise in the two frameworks separately. For example, since the radiative

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