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(Higgs)$_{2,3}$ Quantum Fields in a Finite Volume

I. A Lower Bound*

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Abstract. We consider a Euclidean model of interacting scalar and vector fields in two and three dimensions, and prove a lower bound for vacuum energy in a lattice approximation. The bound is independent of a lattice spacing; it is proved with the help of renormalization transformations in Wilson-Kadanoff form. It extends in principal also to generating functional for Schwinger functions.

1. Formulations of Results, Remarks on the Method, and Notations

The aim of this paper is to give some estimates on the partition function of a lattice approximation of two and three dimensional Euclidean models of interacting scalar and vector fields. These estimates are independent of the lattice spacing. The model is the so-called Proca model, and its (continuous) action is given by

$$S(A, \phi) = \int dx \left[\sum_{\mu=1}^{d} \frac{1}{2} |\partial_\mu \phi(x) + eqA_\mu(x)\phi(x)|^2 + \frac{1}{2} m_0^2 |\phi(x)|^2 + |\lambda| |\phi(x)|^4 + \sum_{\mu,\nu=1}^{d} \frac{1}{4} |F_{\mu\nu}(x)|^2 + \frac{1}{2} \mu_0^2 \sum_{\mu=1}^{d} |A_\mu(x)|^2 \right], \tag{1.1}$$

where ϕ is a scalar field with values in R^N, q is an antisymmetric $N \times N$ matrix, A_μ are components of a vector field and $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$. This model was constructed in the two dimensional case by Brydges et al. [5–7] without any ultraviolet or space cutoffs, including the case $\mu_0^2 = 0$. Here we only prove the ultraviolet stability, however we consider both $d=2$ and 3, and we use a different method, based on a renormalization transformation. We take a lattice approximation for the model as our ultraviolet cutoff. Lattice approximations for gauge field models were introduced by Wilson in [29] and were studied by many authors [5, 6, 11, 16, 17, 22, 27, 28, 30]. The results of Brydges et al. [5, 6] are basic for our paper. They introduced several versions of lattice approximations and they verified their most important properties: physical positivity, diamagnetic in-

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