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Some Results for SU(N) Gauge-Fields on the Hypertorus

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Abstract. We show how to prove and to understand the formula for the "Pontryagin" index P for SU(N) gauge fields on the Hypertorus T^4, seen as a four-dimensional euclidean box with twisted boundary conditions. These twists are defined as gauge invariant integers modulo N and labelled by $n_{\mu\nu}$ $(=-n_{\nu\mu})$. In terms of these we can write $(\nu\in\mathbb{Z})$

$$P=\frac{1}{16\pi^2}\int \mathrm{Tr}(G_{\mu\nu}\tilde{G}_{\mu\nu})d_4x=\nu+\left(\frac{N-1}{N}\right)\cdot\frac{n_{\mu\nu}\tilde{n}_{\mu\nu}}{4}.$$

Furthermore we settle the last link in the proof of the existence of zero action solutions with all possible twists satisfying $\frac{n_{\mu\nu}\tilde{n}_{\mu\nu}}{4}=\kappa(n)=0(\mathrm{mod}\,N)$ for arbitrary N.

1. Introduction

A long standing problem is proving quark confinement in QCD [1]. To simplify the picture a first step in this direction would be to show confinement of static quarks. In this way the problem reduces to an understanding of the behaviour of electric flux strings in quarkless QCD, thus working in pure SU(N) gauge theories, where up to present energies $N=3$. Usually this boils down to studying the behaviour of the vacuum expectation value of the Wilson loop operator [1, 2].

But some time ago 't Hooft [3] introduced another elegant method for studying flux strings. By putting the gauge fields on a four dimensional euclidean box, one can imitate a quark source on one side and an antiquark source on the other side of the box by introducing socalled twisted boundary conditions. These boundary conditions force electric flux into the box, just as gauge invariance forces an electric flux string in-between a quark and antiquark source. Similarly one can introduce magnetic flux in the box, and the great strength of the method is its electric-magnetic duality properties.

Essential is that all fields transform trivially under the centre Z_N of SU(N). Effectively the gauge group is thus SU(N)/Z_N and the twists are labelled by six

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