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Decay of Correlations in the One Dimensional Ising Model With $J_{ij} = |i-j|^{-2}$

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Abstract. A low temperature expansion is constructed for the one dimensional Ising model with Hamiltonian $H = \sum_{i<j} |i-j|^{-2}(1-\sigma_i\sigma_j)$. It is shown that the two point function $\langle \sigma_i ; \sigma_j \rangle$ obeys upper and lower bounds of the form $f(\beta)|i-j|^{-2}$ for inverse temperature β sufficiently large.

Introduction

The one dimensional Ising model with Hamiltonian $H = \sum_{i<j} J(i-j)(1-\sigma_i\sigma_j)$ exhibits a phase transition with spontaneous magnetization at low temperatures if the spin–spin coupling $J(i-j)$ is sufficiently long range. If $J(r) = r^{-\alpha}$ with $\alpha > 2$, there is no magnetization at any temperature [7, 15, 18], but if $\alpha \leqq 2$ there is a transition. The case $\alpha < 2$ was treated by Dyson [7] but the borderline case $\alpha = 2$ was not treated rigorously until Fröhlich and Spencer [10] developed a sophisticated Peierls argument for the model. (Anderson, Yuval, and Hamann [1, 2] studied this case in connection with the Kondo problem and predicted a spontaneous magnetization.) In this paper a more detailed analysis of the borderline case $\alpha = 2$ is made. We obtain precise upper and lower bounds on the long distance behavior of correlation functions at low temperatures.

The technique of Fröhlich and Spencer originated in their study of the two dimensional Coulomb gas and plane rotator models [9]. There it provided a tool to study the long distance behavior of correlation functions when truncation was not necessary to obtain decay. When truncation was necessary, their technique did not apply because it is not a full fledged cluster expansion—it is analogous to Peierls expansion half of a mean field expansion [12]. One of the main motivations of this work is a desire to combine the expansion of Fröhlich and Spencer with

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