Commun. Math. Phys. 85, 481–490 (1982)

Communications in Mathematical Physics

A Proof of the Nielsen–Ninomiya Theorem

D. Friedan*
Service de Physique Theorique, CEN-Saclay, F-91190 Gif-sur-Yvette, France

Abstract. The Nielsen–Ninomiya theorem asserts the impossibility of constructing lattice models of non-selfinteracting chiral fermions. A new proof is given here. This proof fills a technical gap in the two proofs presented by the authors of the theorem. It also serves as prelude to an investigation of the chiral properties of the general lattice model.

1. Introduction

Nielsen and Ninomiya [1, 2] have demonstrated that there can be no net chirality in a lattice model of fermions in which the Hamiltonian satisfies the following conditions:

(1) it is quadratic in the fields;
(2) it is invariant under change of the phase of the fields;
(3) it is invariant under translations of the (cubic) lattice; and
(4) it is local, specifically in the sense that it is continuous in momentum space.

One way of coping with this result is to suppose that chiral fermions on the lattice must be selfinteracting. The Nielsen–Ninomiya theorem then might have a generalization equating the number of chiral fermions to some quantity associated with their interaction. This quantity would have to vanish with the selfinteraction. As a first step towards such a generalization, we reprove the original theorem here, in language potentially adaptable to models with selfinteraction. The structure of the proof suggests that the number of chiral fermions in the general lattice model might be determined entirely from examination of its high energy spectrum.

In [1], the theorem was proved using arguments from homotopy theory (algebraic topology). A second proof [2] called on intersection theory (differential topology). The present proof uses calculus (differential geometry). It is technically more complete than the original proofs. First, it shows the mathematical content to be a part of the theory of characteristic classes. Second, it is easily carried

* address after 1 October 1981: Enrico Fermi Institute, University of Chicago, Chicago, Il 60637

0010-3616/82/0085/0481/$02.00