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Three Body Asymptotic Completeness for $P(\Phi)_2$ Models

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Abstract. We consider weakly coupled even $\lambda P(\Phi)_2$ models that do not have a two-body bound state, and prove asymptotic completeness on the subspace of states with mass between $3m+a(\lambda)$ and $4m-b(\lambda)$, where a and b are positive functions tending to zero with λ. The analytic structure of the six point function, integrated over the three incoming momenta, shows only two Landau singular manifolds (plus normal thresholds) associated to three particle processes.

I. Introduction

The $P(\Phi)_2$ theory has been for almost ten years a mathematically well defined quantum field theory, with energy momentum spectrum such as required for reasonable scattering properties: isolated (cyclic) vacuum and isolated one particle hyperboloid of mass m (Glimm et al. [15]). This guarantees the existence of asymptotic (Fock) spaces $\mathscr{H}^{\text{in}}$ and $\mathscr{H}^{\text{out}}$. A satisfactory interpretation of scattering further requires

$$\mathscr{H}^{\text{in}} = \mathscr{H}^{\text{out}} = \mathscr{H},$$

where $\mathscr{H}$ is the whole Hilbert space of physical states. A complete proof of this property, called "asymptotic completeness," seems at present to be out of reach for any $P(\Phi)_2$ model. The usual approach is to consider subspaces $\mathscr{H}_{(a,b)}$ and $\mathscr{H}^{\text{in,out}}_{(a,b)}$ of states with zero total momentum, and total energy in a given interval (a,b) and to prove

$$\mathscr{H}^{\text{in}}_{(a,b)} = \mathscr{H}^{\text{out}}_{(a,b)} = \mathscr{H}_{(a,b)}.$$

When the interaction polynomial $P(\Phi)$ is even, which we assume, one also distinguishes odd and even subspaces, generated by products of odd and even

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