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On The Spectrum of Schrödinger Operators with a Random Potential

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Abstract. We investigate the spectrum of Schrödinger operators H_ω of the type: $H_\omega = -\Delta + \Sigma q_i(\omega) f(x - x_i + \xi_i(\omega))$($q_i(\omega)$ and $\xi_i(\omega)$ independent identically distributed random variables, $i \in \mathbb{Z}^d$). We establish a strong connection between the spectrum of H_ω and the spectra of deterministic periodic Schrödinger operators. From this we derive a condition for the existence of "forbidden zones" in the spectrum of H_ω. For random one- and three-dimensional Kronig–Penney potentials the spectrum is given explicitly.

Introduction

In this paper we study the spectra of random Schrödinger operators H_ω of the form:

$$H_\omega = -\Delta + \sum q_i(\omega) f(x - x_i + \xi_i(\omega)),$$

where $\{x_i\}_{i \in \mathbb{Z}^d}$ is a Bravais Lattice and $\{q_i\}_{i \in \mathbb{Z}^d}$ and $\{\xi_i\}_{i \in \mathbb{Z}^d}$ are independent, identically distributed random variables. Physically speaking H_ω corresponds to a random "charge"-configuration $\{q_i(\omega)\}$, each $q_i(\omega)$ being located at the random position $x_i - \xi_i(\omega)$ and producing a potential $q_i(\omega) f(x - x_i + \xi_i(\omega))$. Thus H_ω can be used as the Hamiltonian of a model for a "mixed" crystal with centers of strength $q_i(\omega)$ at perturbed lattice positions $x_i - \xi_i(\omega)$ or of a model of a liquid.

Models of this kind were considered by many authors, see for example: Halperin [10], Frisch and Lloyd [7], Luttinger [15], Borland [4], Lieb and Mattis [14] and references therein. Random operators of a more or less different kind are studied e.g. in Pastur [18] and [19], Kunz and Souillard [13], Fukushima, Nagai and Nakao [8], Nakao [17] and references given there.

In [11] the present authors showed that the spectrum of a wide class of random operators, containing the H_ω given above, is a nonrandom set Σ. In the present paper we determine the spectrum of the above operator more precisely.

In the first section we give conditions under which the operator H_ω is well defined and moreover essentially self-adjoint on $C_0^\infty(\mathbb{R}^d)$, the infinitely differentiable

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