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Absence of Discrete Spectrum in Highly Negative Ions

II. Extension to Fermions

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Abstract. We extend the results of [1] to fermions, i.e., we show that if H_N is the Hamiltonian for N electrons in the field of a fixed point charge Z, then there is a constant c such that H_N has no discrete spectrum for $N \geqq N_0 = cZ^{6/5}$.

In a recent paper [1], we showed that the N-particle Coulomb Hamiltonian

$$H_N(W, Z) = -\sum_{j=1}^{N} \Delta_j - \sum_{j=1}^{N} Z r_j^{-1} + \sum_{j<k} W r_{jk}^{-1}$$

has no discrete spectrum for sufficiently large N if we make no permutational symmetry restrictions on the domain $\mathscr{D}(H_N)$. However, we were unable to extend these results to fermions and our extension to bosons was indirect. Sigal [2, 3] has recently proved this result for fermions. In this note, we show how to extend our proof to fermions. Despite Sigal's independent proof, we feel that the discussion which follows is valuable for several reasons. The modifications needed to extend our earlier proof to fermions are very minor. Furthermore, these changes lead to a simplification of the proof given in [1] and also allow us to give a direct proof in the boson case by restricting Ψ to the symmetric domain. In addition, our bound $N_0 \leqq cZ^{6/5}$ for the point at which additional electrons will not bind is better than Sigal's[1].

In what follows we use the notation, equation numbers, etc., of [1] unless otherwise stated. We can summarize the content of this note as: All results of [1] remain valid if $\mathscr{D}(H_N)$, ε_N are replaced by either $\mathscr{D}^+(H_N)$, ε_N^+ or by $\mathscr{D}^-(H_N)$, ε_N^-. Furthermore, in the case of fermions there are constants N_0 and c such that H_N has no discrete spectrum when $N \geqq N_0$ and $N_0 \leqq cZ^{6/5}$. The physical interpretation of this result is that a nucleus with infinite mass and charge Z cannot bind more than N_0 electrons[2].

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1 Although Sigal does not explicitly give an estimate for N_0, the arguments in Sect. III.A and E of [1] can be used to show that his condition $\varrho \sim N^{2/3}$ implies $N_0 \sim Z^2$

2 We have not excluded the possibility of bound states corresponding to eigenvalues embedded in the continuous spectrum

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