Commun. Math. Phys. 85, 309–324 (1982)

Communications in
Mathematical
Physics

Geometric Methods in the Quantum Many-Body Problem. Nonexistence of Very Negative Ions*

I. M. Sigal**

The Weizmann Institute of Science,
Department of Theoretical Mathematics, Rehovot 76100, Israel

Abstract. In this paper we develop the geometric methods in the spectral theory of many-body Schrödinger operators. We give different simplified proofs of many of the basic results of the theory. We prove that there are no very negative ions in Quantum Mechanics.

0. Introduction

In this paper we develop geometric methods for studying the spectral properties of the many-body Schrödinger operators. The adjective "geometric" refers to the basic role played by the analysis of the space configurations of a many-body system in question. To translate this geometry into the quantum-mechanical language, one uses partitions of unity on the state space, L^2 (configuration space). This approach commands a remarkable flexibility. As with trial functions in the variational principle, one can vary and optimize the partitions of unity depending on a problem at hand. This will be demonstrated in the present paper. Moreover, the method is naturally generalizable to operators on manifolds.

The basic property of the Schrödinger operators which permits such an analysis is their locality. Nevertheless, it is remarkable that basically local methods give detailed information about the spectra which are the global characteristics of operators.

The geometric methods in the many-body QM problem first appeared in the pioneering work of Zhislin [Z1]. In the West the geometric ideas come from the works of R. Haag and D. Ruelle on the scattering in the field theory and of Lax-Phillips on the acoustic scattering. The term "geometric methods" was dubbed by B. Simon [Sim1]. The importance of partitions of unity in the QM many-body problem was realized in the classical works of Enss [E] and Simon [Sim1] (see also Deift and Simon [DS]). (For more complete references and detailed comments see [RS3].)

* Partially supported by USNSF Grant MCS-78-01885
** Charles H. Revson Senior Scientist

0010-3616/82/0085/0309/$03.20