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Unitarity Equations and Structure of the S-Matrix at the m-Particle Threshold in a Theory with Pure $m \to m$ Interaction

J. Bros and D. Iagolnitzer

Physique Théorique, C.E.N.-C.E.A. Saclay (Orme), F-91191 Gif-sur-Yvette Cedex, France

Abstract. The structure of the S-matrix at the m-partical threshold $s=(m\mu)^2$ of a $m \to m$ process ($m \geqq 2$) in ν-dimensional space-time is determined in a theory with a simplified unitarity equation corresponding to a pure $m \to m$ interaction. If $(m-1)(\nu-1)$ is odd, a two-sheeted, square-root type structure is obtained as in the usual case of two-particle thresholds in dimension 4. The nature of the singularity is more complicated if $(m-1)(\nu-1)$ is even (e.g. $m=3$ in dimension 4). Results obtained in this case include an orthogonal decomposition of the scattering function T with nonholomic eigenvalues of the form $\left[\frac{1}{2i\pi}\ln\sigma + b_i(\sigma)\right]^{-1}$ [where $\sigma=(m\mu)^2-s$ and b_i is uniform around $\sigma=0$] and a related infinite expansion of T in powers of $\ln\sigma$ involving an on-shell irreducible kernel U which is the analogue for $(m-1)(\nu-1)$ even of Zimmerman's K-matrix.

1. Introduction

While substantial progress has been made in recent years in the analysis of the singularity structure of the S-matrix and of the Green's function of Quantum Field Theory, the knowledge of the exact nature of singularities has however remained limited so far; for instance, whereas the square-root nature of two-particle thresholds is an old result, there is no comparable information on the nature of the three-particle thresholds, even at a heuristic level. The present work gives a treatment of the m-particle threshold and an explicit description of the nature of its singularity in a simplified $m \to m$ scattering theory ($m \geqq 2$) with no subchannel interaction, in arbitrary space-time dimension ν. This treatment is based on the on-shell unitarity-type equation of this simplified theory. Complementary results obtained in the off-shell approach and based on a (simplified) Bethe-Salpeter type equation are described in [1], where the links between the two approaches are explained.

As discussed in [1a] the results accredit the idea that the nature of m-particle thresholds in the actual theory should be determined through an adequate analysis

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