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On a Pathology in Indefinite Metric Inner Product Space

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Abstract. A pathology related to an indefinite metric, which has been pointed out by Ito in connection with construction of a two dimensional quantum field model at a finite cutoff, is mathematically analyzed in a simple model. It is found for a model Hamiltonian with a parameter in an indefinite metric inner product space that eigenvalues with a complete set of eigenvectors changes suddenly from positive integers to negative integers as a parameter crosses a critical value (the Hamiltonian being skew selfadjoint with absolutely continuous spectrum on a pure imaginary axis at the critical value of the parameter), if a fixed (positive definite Hilbert space) topology is used in the completion of the underlying indefinite metric inner product space. However it is also found that if the topology is varied with the parameter of the Hamiltonian in the manner similar to analytic continuation, then the Hamiltonian keeps positive integer eigenvalues with a complete set of eigenvectors.

1. Introduction

It is a great pleasure to dedicate this article to the 60^{th} birthday of Professor Rudolf Haag. In one of his pioneering works on algebraic approach in quantum field theory, Haag gave examples of (bilinear) Hamiltonians whose vacuum vectors give rise to inequivalent representations of canonical commutation relations [1]. In this article, we use similar examples on an indefinite metric Hilbert space to show some pathological phenomena in such a space and a possible method of overcoming such a pathology.

Ito [2] tried to apply the original method of constructive field theory to study the limit of cut-off quantum electrodynamics in one space and one time dimension. The model is exactly solvable in principle by Bogoliubov transformations. Ito works on a Fock space with an indefinite metric and studies what can be considered the vacuum vector for the cut-off Hamiltonian, by applying an appropriate Bogoliubov transformation (mathematically unbounded operators) on the free vacuum vector. It turns out that there is a finite value of the cut-off

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