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Localization in Algebraic Field Theory

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Abstract. The algebra of observables has two distinct local structures. The first, derived from the localization of measurements, gives rise to an additive net structure. The second, derived from the support properties of infinitesimal operations, gives rise to a sheaf structure. It is also shown how an additive net of field algebras acted on by a compact gauge group of the first kind generates an additive net of observable algebras.

0. Introduction

An elementary particle physicist might be defined as a man who, in face of all the evidence, believes in elementary particles. His more theoretically inclined colleague, the quantum field theorist, is forced to comprehend tracks in bubble chambers and irreducible representations of the Poincaré group as compatible aspects of the concept "elementary particle." He will reconcile these aspects using concepts of field theory, which, again in the face of all the evidence, he stubbornly clings to. The ability of "elementary particle" and "field" to survive in the minds of physicists is, in part, their ability to evolve as concepts.

However useful specific quantum fields may prove in analysing specific models, it is unlikely that fields in the sense, say, of unbounded operator-valued distributions, are the basic objects of field theory. Not only are they ill-behaved mathematically but they do not provide an intrinsic description of a physical theory: different fields can describe the same physical theory. It took a man of Rudolf Haag's vision to make a radical break with tradition and provide an alternative view of field theory, algebraic field theory, free of these defects [1]. The basic object is the observable net $\mathfrak{A}$ assigning to a region $\mathcal{O}$ in space-time the algebras $\mathfrak{A}(\mathcal{O})$ generated by the observables which can be measured within $\mathcal{O}$. The connection with fields is that the elements of $\mathfrak{A}(\mathcal{O})$ can be regarded as bounded local functions of the fields, local in the sense that they depend only on the restriction of the fields to $\mathcal{O}$. If fields are introduced at all at this stage, they serve as auxiliary quantities in the construction of the net $\mathfrak{A}$ and one readily understands how different fields can define the same theory. More important is the assertion

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