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Topology of Lattice Gauge Fields*

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Abstract. Non-Abelian gauge fields on a four-dimensional hypercubic lattice with small action density [$\mathrm{Tr}\{1-U(p)\}<0.03$ for SU(2) gauge fields] are shown to carry an integer topological charge Q, which is invariant under continuous deformations of the field. A concrete expression for Q is given and it is verified that Q reduces to the familiar Chern number in the classical continuum limit.

1. Introduction

Differentiable SU(2)[1] gauge fields $A_\mu=-A_\mu^+$ on a four-dimensional torus T^4 (= finite volume with periodic boundary conditions) carry a topological number [1]

$$Q=-\frac{1}{16\pi^2}\int_{T^4} d^4x\,\mathrm{Tr}\{F_{\mu\nu}{}^*F_{\mu\nu}\},$$
$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+[A_\mu,A_\nu]\,;\qquad {}^*F_{\mu\nu}=\tfrac{1}{2}\varepsilon_{\mu\nu\varrho\sigma}F_{\varrho\sigma}, \tag{1}$$

which assumes integer values only. One expects that this structure gives rise to interesting effects at the quantum level, in particular, the topological susceptibility

$$\chi_t=\langle Q^2\rangle/\mathrm{Volume} \tag{2}$$

enters the chiral Ward identities and may be responsible for the large mass of the η' meson (see [2] for a recent discussion).

On a lattice, continuity (in space) is lost and any lattice gauge field $U(n,\mu)$ [3] can be continuously deformed to the trivial field $U(n,\mu)=1$: there is apparently no topological structure. However, this argumentation overlooks the fact that ultimately one is only interested in lattice gauge theories with small values of the bare coupling constant g, i.e. in the region near the (quantum) continuum limit. In

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1 For clarity, the gauge group is taken to be SU(2) throughout the paper. The results can easily be generalized to any compact gauge group

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