Commun. Math. Phys. 84, 515–530 (1982)

Communications in
Mathematical
Physics

Path Integration for Velocity-Dependent Potentials

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Abstract. We extent the prodistribution definition of path integrals to include Lagrangians with velocity-dependent potentials. We use Cameron-Martin transformations to evaluate a large part of a path integral exactly and give techniques for evaluating the remaining terms of the semi-classical expansion of the path integral. The Fredholm determinants, associated with these transformations, are evaluated explicitly in terms of Jacobi matrices defined by the classical system.

Introduction

We develop a method for computing the semi-classical expansion of path integrals for systems having velocity-dependent potentials. Our starting point is the definition of path integrals in terms of prodistributions that was introduced by DeWitt-Morette [1–3]. We extend her methods to include velocity-dependent potentials.

In the semi-classical expansion, the terms in the path integral for a wave function are expanded in a Taylor series around an appropriate classical path. The terms involving the second variation of the action can be combined with the Wiener prodistribution, that defines the path integral, to give a new Gaussian prodistribution that is adapted to integrating the higher order terms of the semi-classical expansion. The use of the new Gaussian prodistribution – or equivalently, the expansion around a classical trajectory instead of a straight line – has the advantage that a larger part of the path integral is evaluated exactly and hence the semi-classical expansion seems to converge faster than the Born expansion. The semi-classical expansion also brings out the connection between the quantum wave function and the family of classical trajectories.

The basic theory of Gaussian prodistributions and their semi-classical expansion is presented in [4]. We extend the results presented there to systems having velocity-dependent potentials. This paper is self-contained; however, many of the results that are derived in [4] are stated here without proof.

0010-3616/82/0084/0515/$03.20