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Representations Obeying the Spectrum Condition

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Abstract. We show that every properly infinite, injective von Neumann algebra acting on a separable Hilbert space is isomorphic to the weak closure of some translation covariant representation, obeying the spectrum condition for the generators of the translation group, of the C^*-algebra of quasilocal observables of a free massless spinor field. We construct explicitly such representations in the case of II_∞ and III_λ factors, $0 < \lambda < 1$.

1. Introduction

The von Neumann algebra generated by a representation π of the C^* algebra $\mathfrak{A}$ of quasilocal observables of a local quantum theory [1] is known to be type I if π is covariant for the space time translation group, the representation $\mathscr{U}$ of this group on $\mathscr{H}_\pi$ fulfills the spectrum condition

$$\mathrm{Sp}(\mathscr{U}) \subset \bar{V}_+ \tag{1.1}$$

and there is a $\mathscr{U}$-invariant vector which is cyclic for π (the vacuum) [2]. In absence of the vacuum, $\pi(\mathfrak{A})''$ is also type I if the spectrum condition (1.1) is sharpened by requiring the existence of a massive particle isolated from the rest of the spectrum [3].

We show that in presence of massless particles all types of von Neumann algebras can appear among the positive energy representations of $\mathfrak{A}$. This answers a question posed by D. Buchholz.

We study a simple model, the even part of the field algebra of a free massless Majorana particle. Specifically, we consider the CAR algebra $\mathfrak{A}(K)$ over K, when K is the direct sum of the Hilbert spaces of the irreducible unitary representations of the covering of the Poincaré group of zero mass, spin 1/2 and helicities $\pm$. The destruction and creation operators $a(f)$, $a(g)^*$, $f, g \in K$, fulfilling the CAR, are related in the standard way to the negative and positive frequency parts of the free massless Majorana field ψ.[1] The local field algebras $\mathfrak{F}(\mathscr{O})$ are the C^*-subalgebras of $\mathfrak{A}(K)$

1 By considering $K \oplus K$ instead of K to allow distinction between particles and antiparticles, we could similarly study a massless Dirac theory

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