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# Hydrodynamics in Two Dimensions and Vortex Theory*

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**Abstract.** We consider the Navier-Stokes equation for a viscous and incompressible fluid in $\mathbb{R}^2$. We show that such an equation may be interpreted as a mean field equation (Vlasov-like limit) for a system of particles, called vortices, interacting via a logarithmic potential, on which, in addition, a stochastic perturbation is acting. More precisely we prove that the solutions of the Navier-Stokes equation may be approximated, in a suitable way, by finite dimensional diffusion processes with the diffusion constant related to the viscosity. As a particular case, when the diffusion constant is zero, the finite dimensional theory reduces to the usual deterministic vortex theory, and the limiting equation reduces to the Euler equation.

## 1. Introduction

In this paper we deal with an incompressible, viscid or inviscid fluid in two dimensions and study the connection between the equations governing the motion of such a fluid and the vortex theory. Furthermore we investigate some aspects of the hydrodynamical equations that, in particular, will suggest a quite natural proof of the existence and uniqueness of the solutions for a wide class of initial conditions.

It is well known that an incompressible and viscous two dimensional fluid, under the action of an external conservative field, is described by the following evolution equations

$$\left.\begin{aligned} &\frac{\partial \omega}{\partial t}(x,t)+(u\cdot\nabla)\,\omega(x,t)-\nu\;\Delta\omega(x,t)=0\,,\\ &\omega(x,t)=\operatorname{curl} u(x,t)=\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2},\\ &\nabla\cdot u=0\,,\end{aligned}\right\} \tag{1.1}$$

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