Commun. Math. Phys. 84, 471–481 (1982)

Communications in
Mathematical
Physics

General Solutions of Nonlinear Equations in the Geometric Theory of the Relativistic String

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Abstract. General solutions for the system of nonlinear equations in the second order partial derivatives with two independent variables are obtained. They determine the basic differential forms of the two-dimensional minimal surface embedded into n-dimensional pseudo-Euclidean space.

1. Introduction

In recent years a good deal of attention has been paid to the derivation of exact solutions for the nonlinear equations in partial derivatives. In elementary particle theory this interest stems from attempts to go outside the limits of perturbation theory in the quantum field approach (solitons, instantons, strings, etc. [1–4]). For the two-dimensional field models the inverse scattering method turned out to be effective for these investigations [5].

A series of papers [6–11] gives geometric interpretation of the nonlinear equations solved by the inverse scattering method. It has been shown that these equations are tightly related with the intrinsic geometry of surfaces in the Euclidean, pseudo-Euclidean and affine spaces (pseudo-spheres, minimal surfaces, surfaces of a constant mean curvature, etc.). Moreover, the linear equations describing the change of the moving basis during the motion of its origin along the surface can be considered as the Lax operators for the relevant nonlinear equation.

In this paper we shall deal with the nonlinear equations describing in differential geometry the minimal surfaces in the pseudo-Euclidean space. The geometric nature of these equations allows one to obtain explicitly their general solutions.

We shall consider two different parametrizations of the minimal surfaces, and as a consequence, we shall obtain two series of the systems of nonlinear equations. The general solutions of these equations will be found exactly. The first series starts with the nonlinear Liouville equation (3.8), then we have the system of two equations (3.16), the new system of three nonlinear equations (3.28), and so on. The other parametrization of the minimal surfaces (the so-called $t=\tau$ gauge) leads to the different series of systems beginning with the D'Alambert equation (4.8).

0010-3616/82/0084/0471/$02.20