Commun. Math. Phys. 84, 439–447 (1982)

Communications in Mathematical Physics

Derivations Vanishing on $S(\infty)$*

Robert T. Powers[1] and Geoffrey Price[2]

1 Department of Mathematics E1, University of Pennsylvania, Philadelphia, PA 19104, USA
2 Department of Mathematics, Indiana University-Purdue University, Indianapolis, IN 46223, USA

Abstract. Let $S(\infty)$ be the group of finite permutations on countably many symbols. We exhibit an embedding of $S(\infty)$ into a UHF-algebra $\mathfrak{A}$ of Glimm type n^∞ such that, if δ is a *-derivation vanishing on $S(\infty)$ and satisfying $\tau\circ\delta=0$, where τ is the unique trace on $\mathfrak{A}$, then δ admits an extension which is the generator of a C^*-dynamics.

1. Introduction

In [4] Goodman showed that if G is a locally compact group, and δ is a closed *-derivation on $C_0(G)$ commuting with the action of G as left translations on the algebra, then δ is a generator of a strongly continuous one-parameter group of *-automorphisms on $C_0(G)$. In a more recent paper, [5], Goodman and Jørgensen consider closed *-derivations on a C^*-algebra $\mathfrak{A}$ commuting with a strongly continuous representation α_G of a compact group G on $\mathfrak{A}$. They define a *-derivation δ to be *tangential* to α_G if it has the aforementioned property (i.e., $\delta\circ\alpha_g=\alpha_g\circ\delta$, for all $g\in G$) and if $\mathfrak{A}^\alpha$, the C^*-algebra of fixed elements of $\mathfrak{A}$, lies in the kernel of the derivation. Under certain restrictions on the system $(\alpha, G, \mathfrak{A})$ (e.g., $\mathfrak{A}$ is abelian, or the action of G on $\mathfrak{A}$ is ergodic) they prove that a derivation tangential to α_G is, in fact, the infinitesimal generator of a strongly continuous one-parameter group of automorphisms.

Suppose now that $\mathfrak{A}$ is a UHF (uniformly hyperfinite) C^*-algebra of Glimm type n^∞: i.e., $\mathfrak{A}=\overset{*}{\bigotimes}_{k\geqq 1} B_k$, where each B_k is a full $n\times n$ matrix algebra over the complex numbers $\mathbb{C}$. Define $S(\infty)$ to be the group of *finite* permutations on the symbols of $\mathbb{N}$, the positive integers. Then there exists a natural embedding of $S(\infty)$ into $\mathfrak{A}$ such that, if G is any compact group, and α_G a strongly continuous representation of *product* type, then $S(\infty)$ lies in the C^*-algebra $\mathfrak{A}^\alpha$ of fixed points of α_G (see [8]). Motivated by the results of [5], we show the following: if δ is a

* Work supported in part by NSF

0010-3616/82/0084/0439/$01.80