

Stationary Solutions of the Bogoliubov Hierarchy Equations in Classical Statistical Mechanics. 4

B. M. Gurevich¹ and Yu. M. Suhov²

1 Department of Mechanics and Mathematics, Moscow State University, Moscow 117234, USSR

2 Institute for Problems of Information Transmission, USSR Academy of Sciences, Moscow 101447 GSP-4, USSR

Abstract. This is the fourth and final paper of a series in which we investigate the stationary solutions of the BBGKY equations. Herein we prove a lemma which forms the basic step in the proof of our Main Theorem characterizing the stationary solutions of these equations which was stated in the first of this series.

Contents

0. Introduction	333
1. Analysis of the Upper Function (An Application of the Upper and Middle Equations) . . .	341
2. Analysis of the Middle Function for $\nu \geq 2$ (An Application of the Middle Equation)	349
3. The One-Dimensional Case.	356
4. Further Study of the Middle Function (An Application of the Middle and Lower Equations)	362
5. Completion of the Proof of the Basic Lemma	370

0. Introduction

0.1. This is the final paper of a series, all bearing the same title (see [1–3]), devoted to characterizing the stationary solutions of the BBGKY hierarchy equations. Our Main Theorem, stated in [1], deals with states of an infinite system of classical particles in R^ν , $\nu \geq 1$. It asserts that the set of those states (within a certain class of states) which correspond to stationary solutions of the BBGKY hierarchy coincides with the set of equilibrium states. As our class of states we take the Gibbs (DLR) states which correspond to potentials (in our terminology, “generating functions”) of a general type (many-body and depending not only on coordinates but on particle velocities as well) which satisfy conditions $(G_1, \mathbf{1}) - (G_6, \mathbf{1})^1$. The hypotheses of our Main Theorem require that the pair interaction potential in the hierarchy satisfies conditions $(I_1, \mathbf{1}) - (I_4, \mathbf{1})$. The condition $(I_4, \mathbf{1})$ restricts the interaction potential to a finite range.

¹ Instead of writing: condition (G_1) from [1], formula (4.1) from [3], etc., we shall write: condition $(G_1, \mathbf{1})$, formula (4.1,3), etc. This convention was adopted in [2] and [3]