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# Decay of Correlations under Dobrushin's Uniqueness Condition and its Applications

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**Abstract.** An estimate on the correlation of functionals of Gibbs fields satisfying Dobrushin's uniqueness condition is given. As a consequence a result of Gross saying that the truncated pair correlation function decays in the same weighted summability sense as the potential can be extended to the whole Dobrushin uniqueness region. Applications to the central limit theorem and the second derivative of the pressure are also given.

## 0. Introduction

The well-known uniqueness theorem of Dobrushin [3] states that there is only one Gibbs state if the interaction is weak which means that the temperature is high or the activity small. This theorem has the advantage of being very general. No condition like finite range, pair interactions, finiteness of the single spin space or translation invariance is needed. Despite its generality the condition is surprisingly sharp as shown by Simon [12]. Moreover one gets not only uniqueness from it, but also properties of the Gibbs state: Dobrushin [4] showed that it is uniformly mixing, and Gross [6], [7] proved results on the decay of correlation and on the differentiability of the pressure. However one of his results, Theorem 2 in [6], was not proved in the whole Dobrushin uniqueness region, and his expression for the second derivative of the pressure in [7] is different from the ususal covariance series. In our paper here we close these two gaps.

In Sect. 2 we recall results from Dobrushin [4] in the form we will need them later. In Sect. 3 we state then our main result on the decay of correlation (Theorem 3.2). As corollaries we get the results of Gross [6] in the whole Dobrushin uniqueness region. In Sect. 4 we apply our results to check known conditions for the central limit theorem for functionals of Gibbs fields, and in Sect. 5 we show that the second derivative of the pressure is equal to the usual covariance series. The main theorems are proved in Sect. 6 by an extension of Dobrushin's uniqueness proof in [4]. We do not construct a dynamics which has the Gibbs state as an invariant measure like in Vasershtein [13] and Gross [6]. Finally in Sect. 7 we

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