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The Phase Transition in the One-Dimensional Ising Model with $1/r^2$ Interaction Energy

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Abstract. We prove the existence of a spontaneous magnetization at low temperature for the one-dimensional Ising Model with $1/r^2$ interaction energy.

1. Introduction, Basic Ideas and Main Results

It has been known for some time that the one-dimensional Ising model exhibits a phase transition when the forces are sufficiently long range. If the interaction energy is given by

$$J(i-j) \equiv J(r) \geqq c\left[\frac{\ln\ln(|r|+3)}{r^2+1}\right],$$

then there is a spontaneous magnetization at low temperature. This result is due to Dyson [2, 4] and was obtained by comparison to a hierarchical model. On the other hand if

$$\lim_{N\to\infty} [\ln(N)]^{-1/2} \sum_{n=1}^{N} J(r)r \to 0,$$

Rogers and Thompson [7] showed that the spontaneous magnetization vanishes for all temperatures. The same result is expected if the exponent 1/2 is replaced by 1. See [3, 8] for other related results.

In this paper we establish a phase transition when $J(r) = 1/r^2$. This is a borderline case which has been discussed by Anderson and Yuval [1] in connection with the Kondo problem. Thouless has also studied this model and predicted a discontinuity in the spontaneous magnetization as a function of temperature—the Thouless

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