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Euclidean Invariance in Statistical Mechanics of Classical Continuous System

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Abstract. It is shown that equilibrium states of classical particles with short range interactions are Euclidean invariant whenever their correlation functions have a clustering which is integrable. The relation between invariance and clustering is analysed for spatial rotations and internal rotational degrees of freedom. The analysis is then extended to the case of long range interactions, including the Coulomb force and jellium systems.

I. Introduction

An equilibrium state which spontaneously breaks a continuous symmetry cannot have an exponentially fast clustering: this is a general formulation of the Goldstone theorem in statistical mechanics. However, to analyse the possible existence of crystalline phases or the existence of phases with orientational order, a more precise formulation of the Goldstone theorem is necessary.

In [1] we have proved a version of the Goldstone theorem for the translation group, namely $\mathscr{L}^1$-clustering states are necessarily invariant under translation. In the present paper, we shall be more specifically concerned with the rotation group; we shall also improve some of the previous results concerning systems with short range interactions. Furthermore, in addition to spatial rotations, we shall also consider systems of particles with internal degrees of freedom, for example classical gas of anisotropic molecules.

We treat separately the case of short range forces (Sect. II) and that of long range forces (Sect. III). In the latter case, which includes N-component plasma and jellium systems (i.e. charged particles with a rigid uniform neutralizing background), the main idea is that the effect of the long range can be taken care by means of sum rules which reflect the shielding property of such systems. [4, 12].

Our definition of equilibrium states and the starting point of our investigation is the BBGKY-hierarchy for the correlation functions. For sufficiently well behaved short range forces it is known that the BBGKY-hierarchy is equivalent to other definitions of equilibrium states such as the classical KMS condition, the Dobrushin–Lanford–Ruelle or the Kirkwood–Salzburg equations. (See [2] and

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