Commun. Math. Phys. 83, 537–561 (1982)

Communications in Mathematical Physics

Construction of Euclidean $(\mathrm{QED})_2$ via Lattice Gauge Theory

Boundary Conditions and Volume Dependence*

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Abstract. Let $\mathscr{v}=\det_{\mathrm{ren}}(1+K_g)$ be the renormalized Matthews-Salam determinant of $(\mathrm{QED})_2$, where $K_g=ieS\not{A}_g$, $S=(\sum\gamma_\mu\partial_\mu+m)^{-1}$ is euclidean fermion propagator of one of the following boundary conditions: (1) free, (2) periodic at $\partial\Lambda$, $\Lambda=[-L/2;L/2]^2$, (3) anti-periodic at $\partial\Lambda$, and $\not{A}_g(x)=(\sum\gamma_\mu A_\mu(x))g(x)$. Here $g(x)=1$ if $x\in\Lambda_0=[-r/2,r/2]^2\subset\Lambda$ and 0 otherwise. Then we show

(i) $\mathscr{v}\in L^p(d\mu(A))$, $p>0$. Further we prove a new determinant inequality which holds for the QED, QCD-type models containing fermions. This enables us to prove:

(ii) $Z(\Lambda_0)=\int\mathscr{v}d\mu(A)\leqq\exp[c|\Lambda_0|]$. Similar volume dependence is shown for the Schwinger functions.

1. Introduction

Several years ago, the author tried to construct $(\mathrm{QED})_2$ taking a basis on a Hamiltonian formalism of QED, where an indefinite metric is explicitly used to ensure the renormalizability. Because of the indefinite matric, however, there are difficulties: for example it is difficult to prove the existence of the vacuum vector [2].

Recently Weingarten [10] proved the integrability of the renormalized Matthews-Salam determinant $\mathscr{v}=\det_{\mathrm{ren}}(1+K_\Lambda)$, where $K_\Lambda=ieS\not{A}$, $S=(\sum\gamma_\mu\partial_\mu+m)^{-1}$ the euclidean fermion propagator which satisfies anti-periodic boundary conditions at $\partial\Lambda$, $\Lambda=[-L/2,L/2]^2$, $\not{A}(x)=\sum\gamma_\mu A_\mu(x)$ and $\{A_\mu(x)\}$ are the euclidean vector fields which satisfy the periodic boundary conditions at $\partial\Lambda$. The anti-periodic boundary condition of S comes from the use of the transfer matrix to prove the diamagnetic inequality. In this work we show the integrability of $\mathscr{v}$ for any one of the following boundary conditions of S and A_μ:

S; free, periodic, anti-periodic boundary conditions,

A_μ; free, periodic, anti-periodic, boundary conditions.

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* Supported by SRC

0010-3616/82/0083/0537/$05.00