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The Evolution of a Turbulent Vortex*

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Abstract. We examine numerically the evolution of a perturbed vortex in a periodic box. The fluid is inviscid. We find that the vorticity blows up. The support of the L_2 norm of the vorticity converges to a set of Hausdorff dimension ~ 2.5. The distribution of the vorticity seems to converge to a lognormal distribution. We do not observe a convergence of the higher statistics towards universal statistics, but do observe a strong temporal intermittency.

1. Introduction

We consider a straight line vortex imbedded in a three-dimensional periodic domain. We perturb the vortex and follow its evolution by a vortex method, in the hope that the calculation will shed light on aspects of the dynamics of vorticity which are significant for the understanding of turbulence.

The equations of motion are Euler's equations. The reasons for assuming that the viscosity is absent are spelled out in [3, 8]: It is reasonable and consistent with both numerical experience and available theory to assume that in a periodic domain the solution of the Navier-Stokes equations converge to the solution of the Euler equations strongly enough for the properties of the energy-containing and inertial ranges to be analyzable in the inviscid case. Such an assumption is implicitly made in Kolmogorov's theory of the inertial range.

The calculations can of course be pursued only for a short time, until the complexity of the flow outstrips the available computer memory and time. However, significant information can be gleaned in this short time. Long time calculations require a rescaling or a renormalization group procedure [8, 22, 29, 30].

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