Commun. Math. Phys. 83, 469–492 (1982)

Communications in
Mathematical
Physics

Renormalization Group Study of a Critical Lattice Model

II. The Correlation Functions

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Abstract. The Renormalization Group is used to study the correlation functions of a nonlocal hierarchical model mimicking the $\lambda(\nabla\phi)^4$ model, dipole gas and the like. It is shown that the infrared behaviour of the correlations is that of the massless gaussian $\frac{1}{2}c(\lambda)(\nabla\phi)^2$.

1. Introduction

In [1] a nonlocal hierarchical model was introduced to mimic the long distance behaviour of the lattice model with Hamiltonian

$$H(\phi)=\sum_x [(\nabla\phi)_x^2+\lambda(\nabla\phi)_x^4].$$

A renormalization group (RG) transformation was defined in finite volume and contractive properties were proved for it uniformly in volume. With the assumption of existence of the thermodynamical limit, the RG was shown to drive the model to a fixed point mimicking the massless lattice field. In this paper we extend the analysis to correlation functions. Using the RG we prove detailed estimates of the long distance behaviour of correlations, showing that in the infrared the model behaves as a massless gaussian lattice field. We also establish the existence of the thermodynamical limit of all correlations and thereby complete the analysis of [1]. In the thermodynamic limit the correlation functions will satisfy convergent (inductive) cluster expansions, generalizations of those working in the massive case now to a massless model. The present paper is selfcontained provided certain results of [1] are taken as given.

Let us briefly recall the model (for motivation, see [1]). We divide the periodic lattice $\Lambda_N=\mathbb{Z}^d_{L^N}$ of volume L^N ($L\in\mathbb{N}$, odd $\geqq 3$) to blocks b^k_y of volume L^{kd} $1\leqq k\leqq N$ centered at yL^k, $y\in\Lambda_{N-k}$ and associate a random variable Z^k_y to the block b^{k+1}_y. Let $\mathscr{A}$ be a function supported on b^1_0 with mean zero, $\mathscr{A}(0)\neq 0$ and nonconstant in $b^1_0\backslash\{0\}$. Denoting

$$z^k_y=\mathscr{A}(y-L[L^{-1}y])Z^k_{[L^{-1}y]} \tag{1}$$

0010-3616/82/0083/0469/$04.80