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A Boson Representation for SU (N) Lattice Gauge Theories

W. Rühl*

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract. SU(N) lattice gauge theories are reformulated in terms of fields varying over non-compact spaces $\mathbb{C}^N$, transforming as N dimensional representations of SU(N) and integrated with Gaussian measure. This reformulation is equivalent to a boson operator representation. Strong coupling expansions based on this formalism do not involve SU(N) vector coupling coefficients.

1. Introduction

In pure Euclidean Yang–Mills field theories on a lattice field, variables range over the group manifold itself. This manifold is compact and a non-trivial Riemannian space. The gauge groups we will consider are SU(N), $N=2,3$ but our results can be immediately generalized to any N. In this article we reformulate such theories in an equivalent fashion in terms of fields taken from the flat non-compact space $\mathbb{C}^N$. They transform as N dimensional representations of SU(N). We will therefore call them "bosonic spinorial variables" for the gauge field. The integration is over a Gaussian measure instead of a Haar measure. A straightforward change of notation leads then to a boson operator formulation of Yang–Mills lattice field theories.

Our approach is based on Bargmann's realization of group representations of SU(N) [1], which makes use of Hilbert spaces of entire analytic functions over $\mathbb{C}^N$ or powers of $\mathbb{C}^N$. This formalism is equivalent to the so-called boson operator calculus [2]. For technical reasons and for the sake of mathematical clarity we prefer to use spaces of analytic functions in this article.

The lattice Λ is assumed to be hypercubic, to have dimension D and the boundary conditions are presumed to be periodic. Let ℓ denote the links and p the plaquettes of Λ. We define the partition function by the standard ansatz

$$Z=\int \prod_{\substack{\text{links}\\ \ell\in\Lambda}} (du_\ell) e^{S(\{u_l\})}.$$

$$u_\ell \in \mathrm{SU}(N) \tag{1}$$

* Permanent address: Fachbereich Physik der Universität Kaiserslautern, D-657 Kaiserslautern, Federal Republic of Germany

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