

Classical and Quantum-Mechanical Systems of Toda Lattice Type. I

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Abstract. The structure of the commutant of Laplace operators in the enveloping and “Poisson algebra” of certain generalized “ $ax + b$ ” groups leads (in this article) to a determination of classical and quantum mechanical first integrals to generalized periodic and non-periodic Toda lattices. Certain new Hamiltonian systems of Toda lattice type are also shown to fit in this framework. Finite dimensional Lax forms for the (periodic) Toda lattices are given generalizing results of Flaschke.

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0. Introduction

In this paper we study the Laplace operator and its commutant on a class of generalized “ $ax + b$ ” Lie groups. This analysis is carried out both in the universal enveloping algebra of the Lie algebra and in the “Poisson algebra”. The problem in the Poisson algebra amounts to finding first integrals for certain Hamiltonian systems (which generalize both the periodic and non-periodic Toda lattices). The

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