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# Nonlinear Stability and Instability of Transonic Flows Through a Nozzle*

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**Abstract.** We study transonic flows along a nozzle based on a one-dimensional model. It is shown that flows along the expanding portion of the nozzle are stable. On the other hand, flows with standing shock waves along a contracting duct are dynamically unstable. This was conjectured by the author based on the study of noninteracting wave patterns. The author had shown earlier that supersonic and subsonic flows along a duct with various cross sections are stable. Basic to our analysis are estimates showing that shock waves tend to decelerate along an expanding duct and accelerate along a contracting duct.

## 1. Introduction

It is well known that gas flows are often highly unstable. This is clear from experimental studies and causes great difficulties in numerical calculations. However, the emergence of shock waves in the flow as a consequence of the nonlinearity of the gas dynamics equations makes it even more difficult for analytical studies. The present paper is a step in understanding such phenomena. We show that for a 1-dimensional model flows along an expanding duct are always asymptotically stable, while flows with a standing shock wave along a contracting duct are dynamically unstable. Since the initial value problem for the gas dynamics equations is expected to be well-posed, by instability we mean that the asymptotic state of a flow does not depend smoothly on the initial state.

The equations which model gas flow in a variable area duct are of the form [1]:

$$\begin{aligned} \frac{\partial \varrho}{\partial t}+\frac{\partial(\varrho u)}{\partial x} &= -\frac{a'(x)}{a(x)}\varrho u, \\ \frac{\partial(\varrho u)}{\partial t}+\frac{\partial(\varrho u^2+p)}{\partial x} &= -\frac{a'(x)}{a(x)}\varrho u^2, \\ \frac{\partial(\varrho E)}{\partial t}+\frac{\partial(\varrho E u+pu)}{\partial x} &= -\frac{a'(x)}{a(x)}(\varrho E u+pu), \end{aligned} \tag{1.1}$$

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