Commun. Math. Phys. 83, 213–242 (1982)

Communications in Mathematical Physics
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On the Regular Holonomic Character of the *S*-Matrix and Microlocal Analysis of Unitarity-Type Integrals

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Abstract. The previously proved results that every analytically renormalized Feynman integral is a regular holonomic function suggests that the *S*-matrix should be locally expressible as an infinite sum of regular holonomic functions. A regularity property *R* is formulated that expresses the condition that the *S*-matrix be locally expressible near each physical point *p* as a convergent sum of regular holonomic functions, with each term enjoying some of the regularity properties of a corresponding Feynman integral. This property *R* holds at every physical point *p* that has yet been analyzed by the methods of axiomatic field theory or *S*-matrix theory. Some analyticity properties of unitarity-type integrals are then examined under the assumption that the *S*-matrix satisfies property *R* and a weak integrability condition. These results rest heavily on some recently proved properties of regular holonomic functions.

1. Introduction

Sato [1] has conjectured that the *S*-matrix satisfies a holonomic system of (micro)-differential equations with characteristic variety determined by the Landau equations. Support for this conjecture has been adduced by Kashiwara and Kawai [2], who have shown that the analytically renormalized Feynman function $F_G(p)$ associated with any Feynman graph G satisfies such a system of equations with characteristic variety confined to the extended Landau variety $\tilde{\mathscr{L}}(G)^{\mathbb{C}}$.

The Feynman functions enjoy an important additional property: they are regular holonomic functions. A regular holonomic function is, by definition, a hyperfunction that satisfies a holonomic system of linear differential equations with regular singularities. Kashiwara and Kawai [3] have developed a microlocal

* Supported in part by NSF MCS 800 6521
** This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. W-7405-ENG-48

0010-3616/82/0083/0213/$06.00