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The Global Existence of Yang–Mills–Higgs Fields in 4-Dimensional Minkowski Space

I. Local Existence and Smoothness Properties*

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Abstract. In this paper and its sequel we shall prove the local and then the global existence of solutions of the classical Yang-Mills-Higgs equations in the temporal gauge. This paper proves local existence uniqueness and smoothness properties and improves, by essentially one order of differentiability, previous local existence results. Our results apply to any compact gauge group and to any invariant Higgs self-coupling which is positive and of no higher than quartic degree.

I. Introduction

This is the first paper in a series of two in which we shall prove the local and then the global existence of solutions to the Yang-Mills-Higgs equations in 4dimensional Minkowski space. In this paper we establish local existence, uniqueness and smoothness properties of Yang-Mills-Higgs fields in the temporal gauge. In the sequel, we shall extend this result to global existence by showing that an appropriate norm of the solutions cannot blow up in a finite time. Our results apply to any compact gauge group and to any invariant, positive Higgs self-coupling of no higher than quartic degree.

Our work on Yang-Mills theory was motivated by an interest in the cosmic censorship conjecture in general relativity. This conjecture states (roughly) that singularities which develop from regular initial data are always hidden inside black holes. Some heuristic arguments given elsewhere by the authors [1,2] strongly suggest that the cosmic censorship conjecture is equivalent to a certain global existence conjecture about the Einstein equations. One hopes to prove the global existence conjecture and thereby to establish the validity of cosmic censorship.

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