Commun. Math. Phys. 83, 123–150 (1982)

Communications in
Mathematical
Physics

The Random Walk Representation of Classical Spin Systems and Correlation Inequalities

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Abstract. Ferromagnetic lattice spin systems can be expressed as gases of random walks interacting via a soft core repulsion. By using a mixed spin-random walk representation we present a unified approach to many recently established correlation inequalities. As an application of these inequalities we obtain a simple proof of the mass gap for the $\lambda(\phi^4)_2$ quantum field model. We also establish new upper bounds on critical temperatures.

0. Introduction

In [1] Symanzik introduced a representation which expressed the ϕ^4 quantum field model as a classical gas of Brownian paths which interact only when they cross. In [2, 3] and in this paper we have developed variants of this formalism which provide a transparent way to establish many inequalities.

In the first two sections we reconsider Symanzik's formalism. We prove two identities. The first identity expresses the spin system as a gas of random walks. In Sect. 3 we use this representation to obtain new upper bounds on critical temperatures. The second identity is a mixed spin-random walk representation. We combine this identity with chessboard estimates [4] and Griffiths [5] inequalities for $N=1$ or 2 component spins to obtain many new and useful results: In Sect. 4, we apply this formalism to show that for a class of classical spin models whose single spin distribution is monotone decreasing, there is always exponential decay of correlations. Hence there is no symmetry breaking. In Sect. 5 we give a new proof of the Lebowitz inequalities [6] and some generalizations related to Newman's Gaussian inequalities [7]. The following section rederives correlation inequalities recently found by Simon, Lieb and Rivasseau [8–10]. The final section of our paper is devoted to a new and elementary proof of the mass gap for the

* Partially supported by N.S.F. Grant No. 79-02490
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**** Partially supported by N.S.F. Grant No. DMR 81-00417

0010-3616/82/0083/0123/$05.60