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Monotonicity of the Free Energy in the Stochastic Heisenberg Model*

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Abstract. The specific free energy of the state at time t of the stochastic Heisenberg model is shown to be non-increasing with t, and to strictly decrease whenever this state is not a Gibbs state of the Hamiltonian. The initial state is assumed to be translation invariant and suitably smooth. For such states a convergence theorem is obtained.

I. Introduction

The classical Heisenberg model is one of a class of lattice spin models in which the range of a single spin is a sphere S^n, $n \geqq 1$, rather than the two-point set S^0, as in the Ising model. The Hamiltonian for these models is given (formally) by:

$$H = -\tfrac{1}{2} \sum_{\substack{x, y \in L \\ |x-y|=1}} \sum \xi(x) \cdot \xi(y), \tag{1}$$

where $\xi(x) \in S^n$ is the spin at site x, L is a d-dimensional lattice, and the "·" denotes dot product in R^{n+1}. Special cases include the planerotor models ($n=1$) and the classical Heisenberg model ($n=2$, and usually $d=3$).

For these models with continuous symmetry groups invariance of phase is expected if $d=2$ and phase-transition with associated continuous symmetry breaking if $d \geqq 3$. These facts were established in [1, 2].

The stochastic Heisenberg model, a probabilistic model for the dynamics of the classical Heisenberg model, was introduced by Faris in [3]. This is a Markovian model with infinitesimal generator

$$\Omega = \Delta - \beta \nabla H \cdot d. \tag{2}$$

In this equation the first term is the infinite-dimensional Laplace operator and generates a Brownian motion of the individual spins. The second term is the inverse

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