Commun. Math. Phys. 83, 83–106 (1982)

Communications in Mathematical Physics

Kowalewski's Asymptotic Method, Kac-moody Lie Algebras and Regularization

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Abstract. We use an effective criterion based on the asymptotic analysis of a class of Hamiltonian equations to determine whether they are linearizable on an abelian variety, i.e., solvable by quadrature. The criterion is applied to a system with Hamiltonian

$$H=1/2\sum_{i=1}^{l} p_i^2+\sum_{i=1}^{l+1}\exp\left(\sum_{j=1}^{l} N_{ij}x_j\right),$$

parametrized by a real matrix $N=(N_{ij})$ of full rank. It will be solvable by quadrature if and only if for all $i\neq j$, $2(NN^T)_{ij}(NN^T)_{jj}^{-1}$ is a nonpositive integer, i.e., the interactions correspond to the Toda systems for the Kac-Moody Lie algebras. The criterion is also applied to a system of Gross-Neveu.

A completely integrable Hamiltonian system in a phase space of dimension $2l$ possesses l independent commuting integrals. Under a compactness condition, the system executes linear motion on an l-dimensional torus defined by these integrals. In most classical cases, the torus is defined by (real) polynomial functions of appropriately chosen phase variables, and the transformation to the separating variables is also algebraic. Moreover, the equations of motion in these new variables are solved by quadrature; it means geometrically that the above torus has an algebraic addition law and that the solutions are straight lines with regard to this law. In most examples, the real torus above is part of a complex torus with algebraic addition law. A Hamiltonian system will be called *algebraically completely integrable* if it can be linearized on an abelian variety (complex algebraic torus with algebraic addition law).

This paper deals with a criterion for algebraic complete integrability, inspired by work of Kowalewski. In celebrated papers [7, 8], she has shown that the *only*

* Supported in part by NSF contract MCS 79-17385
** Supported in part by NSF contract MCS 79-05576

0010-3616/82/0083/0083/$04.80