Commun. Math. Phys. 83, 49–76 (1982)

Communications in
Mathematical
Physics

Feynman Path Integrals and the Trace Formula for the Schrödinger Operators

S. Albeverio[2], Ph. Blanchard[1], and R. Høegh-Krohn[3]*

1 Fakultät für Physik, Universität Bielefeld, Bielefeld Federal Republic of Germany
2 Fakultät für Mathematik, Ruhr-Universität Bochum, Bochum, Federal Republic of Germany
3 Matematisk Institutt, Universitetet i Oslo, Oslo, Norway

Abstract. We study Schrödinger operators of the form $H = -\frac{h^2}{2}\Delta + \frac{1}{2}x\cdot A^2 x + V(x)$ on $\mathbb{R}^d$, where A^2 is a strictly positive symmetric $d\times d$ matrix and $V(x)$ is a continuous real function which is the Fourier transform of a bounded measure. If λ_n are the eigenvalues of H we show that the theta function $\theta(t) = \sum_n \exp\left(-\frac{i}{h}t\lambda_n\right)$ is explicitly expressible in terms of infinite dimensional oscillatory integrals (Feynman path integrals) over the Hilbert space of closed trajectories. We use these explicit expressions to give the asymptotic behaviour of $\theta(t)$ for small h in terms of classical periodic orbits, thus obtaining a trace formula for the Schrödinger operators. This then yields an asymptotic expansion of the spectrum of H in terms of the periodic orbits of the corresponding classical mechanical system. These results extend to the physical case the recent work on Poisson and trace formulae for compact manifolds.

1. Introduction

The study of the relations of quantum mechanics and classical mechanics goes back to the very origin of quantum physics, i.e. to the years where the "old quantum theory" of Bohr, Einstein, Sommerfeld was developed and was to lead to what is now understood as quantum mechanics (Heisenberg, Schrödinger 1925–1926). We have discussed before [2], by means of our definition of Feynman path integrals [3] the way in which the solutions of the quantum equations of motion are connected to classical motions in the precise sense of asymptotic expansion in powers of Planck's constant.

In the present paper we are concerned with the relation of the eigenvalue

* Partially supported by the USP-Mathematisierung, University of Bielefeld (Forschungsprojekt "Unendlich dimensionale Analysis")

0010-3616/82/0083/0049/$05.60