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On Cantoni's Generalized Transition Probability

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Abstract. We obtain simple expressions of the "generalized transition probability" proposed by V. Cantoni, for both classical and quantum mechanics. We compare the result with the ordinary quantum mechanical transition probability.

I. Introduction

Using Mackey's axiomatization of physical theories ([1], p. 63), V. Cantoni introduced a function $T(\alpha, \beta)$ defined on pairs of states α, β. When the physical theory in question is quantum mechanics and for the special case of pure states, Cantoni proved that $T(a, \beta)$ equals the "transition probability" $|(\alpha, \beta)|^2$. In that sense, he named the function T "generalized transition probability."

Our main purpose in this work is to give simpler expressions of T for both classical and quantum theories since Cantoni's definition is quite involved. We shall prove furthermore that in the quantum case, $T(\alpha, \beta)$ equals the quantum mechanical "transition probability" between the states α and β, each time that this concept has an unambiguous sense.

We now recall that in Mackey's system, one considers a set $\mathcal{O}$ of observables and a set $\mathcal{S}$ of states. For each $A \in \mathcal{O}$ and $\alpha \in \mathcal{S}$, and for any Borel subset E of the real line $\mathbb{R}$, one denotes by $p(A, \alpha, E)$ the probability that a measurement of A performed on a system in the state α will yield a result lying in E. Accordingly, $\alpha_A(E) \equiv p(A, \alpha, E)$ is a probability measure on $\mathbb{R}$. Mackey then imposes some axioms involving $\mathcal{O}, \mathcal{S}$ and the probability p. We shall make use only of the first three, which trivially hold in all known physical theories.

Cantoni's definition now runs as follows: for any pair of states α, β and any observable A, define the expression $T_A(\alpha, \beta)$ by

$$T_A^{1/2}(\alpha, \beta) = \int_{\mathbb{R}} \left(\frac{d\alpha_A}{d\sigma} \frac{d\beta_A}{d\sigma} \right)^{1/2} d\sigma, \tag{1}$$

where σ is any measure with respect to which both α_A and β_A are absolutely continuous. It is easy to see that T_A is independent of σ. Finally define

$$T(\alpha, \beta) = \inf_{A \in \mathcal{O}} T_A(\alpha, \beta). \tag{2}$$

This is Cantoni's "generalized transition probability" [2, 3, 4].

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