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# Removable Singularities in Yang–Mills Fields

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**Abstract.** We show that a field satisfying the Yang–Mills equations in dimension 4 with a point singularity is gauge equivalent to a smooth field if the functional is finite. We obtain the result that every Yang–Mills field over $R^4$ with bounded functional ($L^2$ norm) may be obtained from a field on $S^4 = R^4 \cup \{\infty\}$. Hodge (or Coulomb) gauges are constructed for general small fields in arbitrary dimensions including 4.

There has been a great deal of mathematical interest in the topological and geometrical methods used to construct the instanton solutions to the Yang–Mills equations [1–3]. More recently several articles treating analytic properties have appeared [6], [8], [10], [14]. We consider properties of the Euclidean (Riemannian, elliptic) equations and derive some standard *a priori* estimates on solutions. The main result is a local regularity theorem in 4-dimensions: A Yang–Mills field with finite energy cannot have isolated singularities if its structure group is compact. Apparent point singularities, *including singularities in the bundle*, may be removed by a gauge transformation. In particular, a Yang–Mills field on a bundle over $R^4$ extends to a smooth field on a bundle constructed over $R^4 \cup \{\infty\} = S^4$.

For convenience we concentrate on bundles over flat manifolds. For the regularity theory, the curvature of the manifold itself is not particularly important. In this paper we also assume all solutions have smooth curvatures where they are defined. Other references have handled the question of weak solutions in detail [10, 14, 16]. An announcement of the results in this paper has appeared [15] and an outline of the proof also appears in [6]. Parker has generalized these results to coupled systems in 4 dimensions.

We give a brief description of the problem in Sect. 1 to establish our notation. In Sect. 2, we prove a number of tedious technical lemmas on canonical gauges for fields with small curvatures which are necessary later. Standard *a priori* estimates appear in Sect. 3. The proof of the removability of singularities in Sect. 4 is remarkably similar to the proof of the removability of singularities of harmonic maps contained in [12].

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