

Unbounded Derivations Commuting with Compact Group Actions*

F. Goodman¹ and P. E. T. Jorgensen²

¹ Department of Mathematics, University of Pennsylvania, Philadelphia, PA, 19104, USA

² Mathematics Institute, Aarhus University, Ny Munkegade, DK-8000 Aarhus C, Denmark

Abstract. Let δ be a closed $*$ derivation in a C^* algebra \mathfrak{A} which commutes with an ergodic action of a compact group on \mathfrak{A} . Then δ generates a C^* dynamics of \mathfrak{A} . Similar results are obtained for non-ergodic actions on abelian C^* algebras and on the algebra of compact operators.

1. Introduction

In [9] Sakai showed that a non-zero closed $*$ derivation in $C(\mathbb{T})$ commuting with translations is a constant multiple of the derivative. Following this, it was shown in [4, 7] that if G is a locally compact group and δ is a closed $*$ derivation in $C_0(G)$ commuting with left translations by elements of G , then δ is the generator of a C^* dynamics (i.e., strongly continuous one-parameter group of $*$ automorphisms) of $C_0(G)$. A like result holds for $C_0(G/H)$ (H a closed sub-group of G), provided that G is either separable or the projective limit of Lie groups. In this note, we assume G is compact and we obtain similar results for an ergodic action of G on an arbitrary C^* algebra (Theorem 2.1), and for an arbitrary action of G on an abelian C^* algebra (Theorem 3.2), or on the algebra of compact operators (Theorem 4.1).

We refer to [1, 3, 9] for background on unbounded derivations in C^* algebras.

2. Ergodic and G -Finite Actions

Before stating our first result we recall a few facts about Banach space representations of compact groups. Let V be a Banach space, G a compact group, and α a strongly continuous representation of G on V . For each $\pi \in \hat{G}$, define $P_\pi: V \rightarrow V$ by

$$P_\pi(x) = \int_G \dim(\pi) \operatorname{tr}(\pi(s)) \alpha_s(x) ds.$$

* Research supported by N.S.F.