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Mean Field Approximation is Exact in the Many-Component Limit of Potts Lattice Gauge Model

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Abstract. The statement in the title concerning free energy is proved for a version of mean field approximation with previously fixed temporal gauge.

1. Introduction

Mean field theory yields a useful qualitative insight into a phase structure of lattice gauge theories and it is believed to be reliable for sufficiently high dimensions of spacetime [1]. Indeed, some of its predictions were confirmed by Monte Carlo experiments [2]. On the other side the use of mean field theory for lattice gauge models remains to be rather suspicious: the gauge invariance is broken in its very formulation—an order parameter is introduced that is known to vanish in a lattice gauge theory [3].

Though believed to be accurate in the limit of infinite dimensions, the exactness of a mean field approximation in this limit has, to the author's knowledge, never been proved*. Here we address a similar problem, but we investigate a many component limit instead. Namely, we consider a q-state Potts lattice gauge model [4, 5] in the limit $q \to \infty$ and show that the mean field calculation of its free energy becomes exact. A similar result was proved recently for the conventional Potts model by Pearce and Griffith [6]. The remarkable feature of the gauge generalization of the Potts model is that a mean field approximation is exact in the limit $q \to \infty$ only when formulated in a variant that amounts to fixing timelike variables and applying mean field only to remaining links. To the contrary when mean field is applied directly to all links, without previous gauge fixing, one gets a strictly larger free energy and a mean field approximation fails to be exact. Thus, contrary to the case of the infinite dimensions limit [1], the differences between these two formulations of a mean field theory are not washed out in the limit $q \to \infty$. The situation is in accordance with a feeling that a (gauge noninvariant) mean field approximation should match only a model with a previously broken gauge symmetry.

A gauge Potts model as well as corresponding mean field theories will be introduced in detail in the next section. Here we only state our main result:

* See Note added in proof

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