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Phase Diagrams and Cluster Expansions for Low Temperature $\mathscr{P}(\phi)_2$ Models*

II. The Schwinger Functions

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Abstract. We give a cluster expansion for the Schwinger functions of the stable phases found in Part I. The Wightman axioms, the mass gap, and asymptoticity of perturbation theory follow.

In Part I the phase diagram of a generic low temperature $\mathscr{P}(\phi)_2$ quantum field model was mapped out. At each point in the diagram a number of stable phases q_0 were found such that

$$\frac{Z(\mathbb{V}^q)}{Z(\mathbb{V}^{q_0})} \leqq e^{2\lambda^{1/2}|\partial\mathbb{V}|}$$

for every q. We now use this information with some other Part I machinery to give a cluster expansion for the Schwinger functions in the stable phases. We also prove the convergence estimates needed in Parts I and II. The reader is referred to the list of references in Part I.

4. An Expansion for the Schwinger Functions

4.1. Constrained Expansions

In this chapter we derive a convergent expansion for the Schwinger functions from bounds on ratios of partition functions. The presence of clusters containing field monomials introduces constraints on partition function sums. The constraints must be handled in such a way that the phase structure of Chap. 3 is not destroyed.

So far we have always multiplied clusters by ratios of interior partition functions. This procedure must be altered for clusters surrounding squares containing field monomials. In [20], *a priori* bounds on ratios of partition functions in non-simply-connected regions were available. Thus it was possible to multiply $\tilde{\varrho}(\mathbb{Y})$ by a ratio of partition functions in $(\mathbb{I}\mathrm{nt}\,\mathbb{Y})\setminus\mathbb{X}$, where $\mathbb{X}$ is a cluster containing field monomials.

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