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Phase Diagrams and Cluster Expansions for Low Temperature $\mathscr{P}(\phi)_2$ Models*

I. The Phase Diagram

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Abstract. Low temperature phase diagrams of two-dimensional $\mathscr{P}(\phi)$ quantum field models are constructed. Let $\mathscr{P}$ lie in an $(r-1)$-dimensional space of perturbations of a polynomial with r degenerate minima. Perform a scaling $\mathscr{P}(\phi)\to\lambda^{-2}\mathscr{P}(\lambda\phi)$ and assume $\lambda\ll 1$. We construct k distinct states on $\binom{r}{k}$ hypersurfaces of codimension $k-1$ in the space of perturbations. An expansion is used to exhibit exponential clustering of the Schwinger functions of each of these states. At the core of the construction is a general technique for finding the thermodynamically stable phases from a collection of competing minima. We draw on ideas of Pirogov and Sinai [24] for this problem.

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