Commun. Math. Phys. 82, 257–260 (1981)

Communications in Mathematical Physics

Comment on "Analytic Scattering Theory for Many-Body Systems Below the Smallest Three-Body Threshold"

E. Balslev

Matematisk Institut, Universitetsparken, Ny Munkegade, DK-8000 Aarhus C, Denmark

Abstract. The proof of [1, Lemmas 2.1–2.3] is completed, showing that the operators of multiplication by k^2 in $H^{t,\ell}$, $|t| \leqq 1$, $\ell = 0, \pm 2$, have spectrum $\bar{R}^+$ and generate the holomorphic semigroups $e^{\zeta k^2}$, $\operatorname{Re}\zeta < 0$.

It is pointed out, that [1, (5.54)] does not hold. Accordingly, a new version of [1, Theorem 5.15] is proved, saying that (5.44) defines an isomorphism of $\tilde{\mathcal{N}}(G_+(z,\kappa))/\tilde{\mathcal{N}}_0(G_+(z,\kappa))$ onto $\mathcal{N}(\mathcal{S}_\lambda^*(\bar{z}))$.

1. On the Proof of Lemmas 2.1–2.3

Lemma 2.0. *The operators $\tilde{H}_0$ defined by multiplication by k^2 in the spaces $H^{t,\ell}$ for $t = \pm s$, $0 \leqq s \leqq 1$, $\ell = 0, \pm 2$ with domains $H^{t,\ell+2}$ have the spectrum $\bar{R}^+$ and generate holomorphic semigroups $e^{\zeta \tilde{H}_0}$ defined for* $\operatorname{Re}\zeta < 0$.

Proof. Clearly, $(k^2 - z)^{-1} \in \mathcal{B}(H^{1,\ell}) \cap \mathcal{B}(H^{0,\ell})$, $\ell = 0, \pm 2$, $z \notin \bar{R}^+$, and hence by interpolation $(k^2 - z)^{-1} \in \mathcal{B}(H^{s,\ell})$, $0 < s < 1$, $\ell = 0, \pm 2$.

By duality, $(k^2 - z)^{-1} \in \mathcal{B}(H^{-s,-\ell})$. Obviously, $(k^2 - z)^{-1}$ is unbounded in any of these spaces for $z \in \bar{R}^+$, hence $\sigma(\tilde{H}_0) = \bar{R}^+$.

$\tilde{H}_0$ generates the semigroup $\tilde{\mathcal{U}}(\zeta)$ given by

$$\tilde{\mathcal{U}}(\zeta) = e^{\zeta k^2} \quad \text{in} \quad \mathcal{B}(H^{s,\ell}), \qquad \operatorname{Re}\zeta < 0.$$

Clearly, $\tilde{\mathcal{U}}(\zeta)$ is a uniformly bounded semigroup. For $s = \ell = 0$, $\zeta < 0$, $\tilde{\mathcal{U}}(\zeta)$ is the bounded semigroup $e^{\zeta H_0}$ generated by the self-adjoint operator H_0 in $\mathcal{L}^2$. Thus, for $f \in H^{0,2}$

$$t^{-1}(e^{-tk^2} - 1)f \xrightarrow[t \to 0]{} -H_0 f \quad \text{in } \mathcal{L}^2, \text{ hence in } H^{-s}. \qquad (*)$$

From this it follows that $(*)$ holds for all $f \in \mathcal{D}(\tilde{H}_0)$, and it is easy to see that the operator $\tilde{H}_0$ in $H^{-s,0}$ is the infinitesimal generator of $\tilde{\mathcal{U}}(\zeta)$.

A similar argument proves the same for $\tilde{H}_0$ in $H^{-s,-2}$, and using the duality of H^s with H^{-s} and $H^{s,2}$ with $H^{-s,-2}$ the same is proved for $\tilde{H}_0$ in H^s and $H^{s,2}$.

0010-3616/81/0082/0257/$01.00