Higher Local Conservation Laws
for Nonlinear Sigma Models on Symmetric Spaces

Harald Eichenherr[1] and Michael Forger[2],[*]

1 CERN – Geneva, CH-1211 Geneva 23, Switzerland
2 Fakultät für Physik der Universität Freiburg, D-7800 Freiburg i. Brsg., Federal Republic of Germany

Dedicated to the Memory of our Colleague and Friend Jorge André Swieca

Abstract. We show that for the classical two-dimensional nonlinear σ-model on a Riemannian symmetric space of dimension m and rank p, there exist p independent series of higher local conservation laws, and we reduce the field equations of the model to a system of nonlinear partial differential equations possessing an associated Lax pair and involving $m + p$ independent variables.

1. Introduction

This is the third in a series of papers devoted to analyzing the structure of classical two-dimensional nonlinear σ-models on Riemannian symmetric spaces $M = G/H$. These are field theories of geometric nature which generalize the nonlinear σ-models on the spheres $S^{N-1} = SO(N)/SO(N-1)$ [1] or the complex projective spaces $\mathbb{C}P^{N-1} = SU(N)/S(U(1) \times U(N-1))$ [2]. Our current interest in them mainly stems from the possibility to study systematically the differential geometric roots of integrability properties.

In our first paper [3, I], we introduced a general formulation of the nonlinear σ-model on a Riemannian homogeneous space (see also [4, 5]), and we proved it to possess the so-called *dual symmetry* if (and only if – cf. [6]) the homogeneous space is a symmetric one. This hidden dynamical symmetry yields a linear Lax representation of the nonlinear field equations and leads to an infinite series of *nonlocal* conservation laws, thus generalizing the corresponding well-known features of the S^{N-1} prototype model [7] or the $\mathbb{C}P^{N-1}$ model [2].

In our second paper [3, II], we used certain structural properties of symmetric spaces to clarify the relation between the principal nonlinear σ-model on a Lie group G and the nonlinear σ-models on the symmetric quotient spaces $M = G/H$ for G, and to show that instantons are fixed points of the dual symmetry.

Our present work deals with the derivation of higher *local* conservation laws for the nonlinear σ-model on an arbitrary Riemannian symmetric space $M = G/H$, generalizing the corresponding well-known higher local conservation laws for the