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Bounds on the Coupling Constant in Two Dimensional Boson Models

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Abstract. We consider two dimensional boson field theories with an interaction potential $\mathscr{V}(\phi)$. We show how to define a cut-off, renormalised Hamiltonian for a certain class of non-polynomial $\mathscr{V}(\phi)$, which are defined via an integral transform. We formulate precisely a variational argument devised by Coleman, obtaining a constraint on the coupling constant of the theory with general $\mathscr{V}(\phi)$, and illustrate the argument with several examples.

1. Introduction

In this note we will consider boson field theories obtained by quantising the classical system with Lagrangian density

$$\mathscr{L} = -\tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \lambda^{-2}\mathscr{V}(\lambda\phi)$$

for a range of potentials $\mathscr{V}$. The Hamiltonian density corresponding to $\mathscr{L}$ is

$$\mathscr{H} = \mathscr{H}^0 + \lambda^{-2}\mathscr{V}(\lambda\phi),$$

with

$$\mathscr{H}^0 = \tfrac{1}{2}[\pi^2 + (\nabla\phi)^2],$$

where π is the field conjugate to ϕ. We will allow only two space-time dimensions. Our aims are two-fold, namely:

(1) to define a cut-off, renormalised Hamiltonian for a range of non-polynomial interactions $\mathscr{V}$;

(2) to reformulate within the framework of constructive field theory a variational argument due to Coleman [1], which places constraints on the various parameters appearing in $\mathscr{V}$.

Let us begin by reviewing the argument which Coleman applied to the sine-Gordon model, defined by the potential

$$\mathscr{V}(\phi) = k(1 - \cos\phi), \qquad k > 0.$$

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