

# Borel-Summability of the High Temperature Expansion for Classical Continuous Systems

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**Abstract.** It is shown that for classical gases with stable, bounded and absolutely integrable pair interactions, the Taylor expansions in  $\beta$  of the correlation functions and the pressure are Borel-summable at  $\beta = 0$ .

## 1. Introduction

The question of analyticity in  $\beta$  for classical continuous systems was considered some years ago by Lebowitz and Penrose [1]. Among other results they showed that for hard core potentials pressure and correlation functions are analytic at  $\beta = 0$ . In this paper we treat the case of bounded potentials, where analyticity is not to be expected, as the expansion is around the ideal gas and the negative of a stable potential is unstable, which causes divergence of the pressure for negative  $\beta$  in the finite volume.

## 2. Infinite Volume Correlation Functions

We assume the interaction potential  $\phi$  to satisfy stability,

$$\sum_{\substack{i,j=1 \\ i < j}}^m \Phi(x_i - x_j) \geq -mB \quad \text{for some constant } B \quad (1)$$

and

$$\|\Phi\|_{\infty} < \infty, \quad (2)$$

$$\|\Phi\|_1 < \infty. \quad (3)$$

Eqs. (2) and (3) imply regularity ([2], ch. 4.1):

$$\int |e^{-\beta\Phi(x)} - 1| dx = C(\beta) < \infty \quad \text{for } \beta \in \mathbb{C}. \quad (4)$$

We shall use the representation of the correlation functions given by Ruelle ([2], ch. 4.2.):

On the Banach-spaces  $E_{\xi}$ ,  $\xi > 0$  of sequences of complex functions  $\varphi =$